Sample size for a Ballot Paper Scanning Assurance Plan for ACT Legislative assembly elections

Executive summary

The Australian Bureau of Statistics (ABS) has agreed to work with Elections ACT on the design of a sampling scheme for the auditing of scanned paper ballots. The auditing clerically checks a sample of scanned ballot papers to confirm the vote has been correctly captured through the scanning process. The ABS has previously assisted the Australian Electoral Commission (AEC) with a sampling scheme for the auditing of Senate ballot papers.

The ABS proposes a sample size of 1 in 90 scanned ballot papers. There are expected to be approximately 92,000 scanned ballot papers in the 2024 ACT Legislative Assembly election, in which case a 1 in 90 sample will deliver a sample size of 1022 ballots. This sample size is slightly larger than the sample size used by the AEC within the ACT for Senate elections, and under the anticipated outcome of zero errors detected in the audit sample, will allow Elections ACT to state there is 99% confidence that the number of errors in the full population of ballot papers is smaller than 0.009 the average size of an electorate quota, or 99% confidence that there are no more than 82 erroneous ballot papers on average within an electorate (i.e. no more than 409 errors across all 5 electorates). This report also assesses options of a 1 in 200 sample (sample size of approximately 460), and a 1 in 60 sample (sample size of approximately 1500).

The use of a sampling rate (e.g. 1 in 90) instead of a set sample size (e.g. a sample of 1000) means the sample will provide the same confidence limit for the number of errors even if the total population size of scanned ballot papers changes. That is, if zero errors are detected in the audit sample, then the recommended sampling scheme gives 99% certainty that the number of errors is not more than 409 even if the total number of ballot papers is as small as 50,000 or as large as 120,000. However, there are also advantages to using a set sample size of say 1000 to ensure a sample size does not fall below what may be perceived as defensible amount. There is little difference in practice between using a set sampling rate or a set sample size approach.

Key elements of a ballot paper scanning audit

A paper "Assessing the accuracy of the Australian Senate count" by Michelle Blom, Philip B. Stark, Peter J. Stuckey, Vanesse Teague and Damjan Vukcevic, May 31, 2022,[1] suggests an election audit should encompass the following characteristics.

- Provide sufficient evidence that the error rate is low enough that the results deserve to be trusted.
- Estimate the average number of errors per ballot and the percentage of ballots that have at least one error separately in each electorate.
- The sequence of ballots to be audited needs to be chosen transparently and unpredictably, e.g. through use of a random seed and an algorithm, committed to in advance, that transforms the random seed into a sequence of ballot papers.

The approach proposed in this report is consistent with the advice contained in the paper by Blom et al. The first point, that the error rate is low enough, is difficult to establish objectively in advance of

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[1] https://arxiv.org/pdf/2205.14634.pdf

an election as is recognized in the paper. The suggestion is to establish (or to estimate) how many votes would need to change to change the outcome of an election, but this can only be done after the election is held. In the even that an election outcome is decided with a margin of very few votes, there is the option to conduct additional auditing to increase the total audit sample size and reduce the confidence limit for the number of errors to below the margin that could change the result. The sampling scheme options discussed in this report provide a high level of confidence that the error rate within an electorate is of the order of 0.01 quotas, assuming that a very small number of errors are detected in the audit sample.

On the second point, using a set sampling rate (or alternatively allocating a set sample proportionately across electorates in line with the total number of scanned ballots within the electorate) makes it very easy to estimate the average number of errors per scanned ballot (the sample average is used) and the percentage of scanned ballot papers with at least one error (the percentage within the sample is used).

On the third point, Elections ACT have existing processes to generate a random seed and to use this to select a sample of ballot papers, this meets the principle expressed in the paper by Blom et al. The paper discusses a method of geometric skipping designed to ensure that every possible combination of papers with a set sampling probability can be realized, but while suggested this approach is presented as just one possible method. The method would add additional complexity to the existing Elections ACT method of selecting a set sample size with a batch of ballot papers for negligible benefit. The method of selecting 5 ballot papers for audit with a random start from each batch that is randomly selected for audit is statistically sound, widely used and highly defensible.

Use of a set sampling rate

An appropriate sampling scheme can be designed to either use a set sampling rate (e.g. select 1 in every 90 ballot papers) or a set sampling size (e.g. a sample of 1000 ballot papers). There is little difference in terms of performance, as if the total number of ballot papers is known then a sampling rate can be set to give a specified sample size, or vica versa. For example, if there are 92,000 ballot papers in total, then a sampling rate of 1 in 92 can be used to give a sample size of 1000 or a sample size of 1022 can be used to give a sampling rate of 1 in 90. There are advantages and disadvantages of the different approaches for the operations of the auditing.

Advantages of a fixed sample size:

- Sample size is fixed and so the resourcing required for the auditing is known.
- A fixed sample size will give a fixed accuracy for the estimates of error rate, e.g. for a fixed sample size of 1000 there is 95% confidence that the error rate is not more than 0.30% when 0 errors are detected, regardless of the number of ballot papers in the population.

Advantages of a fixed sampling rate:

- A fixed sampling rate will give a fixed upper limit for the number of ballot papers with a scanning error, e.g. for a fixed sampling rate of 1 in 90, there is 95% confidence that not more than 267 ballot papers have a scanning error when 0 errors are detected in the sample, for population sizes of 76,000 ballot papers and higher. There is only a small difference for small population sizes, e.g. if there are only 20,000 ballot papers, then a 1 in 90 sample will give a 95% confidence limit of 265 errors.
- A fixed sampling rate scheme helps to ensure all ballot papers have the same chance of being selected for audit – there is an even rate applied across all electorates, locations and all days in which the auditing in undertaken.

If the total number of votes to be scanned can be accurately estimated, then the impact of these advantages is reduced compared to situations where the total number of ballot papers varies by a greater amount, that is, there is little practical difference between the approaches.

In this report options for sample size are presented as sampling rates, with the corresponding sample size based on a population of 92,000 paper ballots. The use of a sampling rate is proposed by the paper by Blom et al (section 2.2).

Options for sample size

Sample of 1 in 200 scanned ballot papers (sample size of approximately 460 papers)

This option is suggested as the minimum viable audit sample size and can be sufficient to say with confidence that the quality of the scanning is comparable to the quality of scanning of ACT Senate papers for Federal elections. The option can be implemented by selecting 1 in every 20 batches of ballot papers, and then selecting 1 in every 10 ballot papers from within the batch (i.e. 5 papers from each batch).

The AEC samples 1 in 300 senate papers in the ACT to quality assure the ballot paper scanning process, however there is no option for electronic voting, so there is a greater total population of papers in a federal senate election. For the 2022 senate election, 290,308 votes were cast in the ACT, a 1 in 300 sample then gives a total sample size of 969 ballot papers selected for audit. The sample size under this option is less than half the sample size used for senate elections.

The AEC senate process was designed to provide 95% confidence that the senate scanning error rate is lower than 0.81% (or not more than 2351 errors in the population of 290,308), and 99% confidence that the error rate is lower than 0.95% (not more than 2757).

If this sampling option is used for the 2024 ACT Legislative assembly election, then if:

- *0 errors are detected in sample*, there is 95% confidence the error rate is lower than 0.65% and 99% confidence it is lower than 1.00%. In a population of 92,000 scanned voted this means 95% confidence there are not more than 595 erroneous ballot papers and 99% confidence there are not more than 913. In this scenario the 99% confidence limit for the error rate is slightly higher than the confidence interval for senate scanning, although the number of ballot papers that could have errors is substantially smaller, due to the smaller population of scanned ballot papers. (If a sample of 1 in 190 is selected instead of 1 in 200 then the 99% limit for the error rate will match the AEC value of 0.95%).
- *1 error is detected in sample*, there is 95% confidence the error rate is lower than 1.03% and 99% confidence it is lower than 1.44%. In a population of 92,000 scanned voted this means 95% confidence there are not more than 942 erroneous ballot papers and 99% confidence there are not more than 1315. In this scenario the confidence limits for the error rate exceed the AEC limits although the confidence limits for the number of erroneous ballot papers are lower than for the AEC.
- *2 errors are detected in sample*, there is 95% confidence the error rate is lower than 1.36% and 99% confidence it is lower than 1.82%. In a population of 92,000 scanned voted this means 95% confidence there are not more than 1250 erroneous ballot papers and 99% confidence there are not more than 1663. In this scenario the confidence limits for the error rate exceed the AEC limits although the confidence limits for the number of erroneous ballot papers are lower than for the AEC.

As no errors in scanning were detected in three previous Legislative Assembly elections, with a combined audit sample size of 9000, it is unlikely that errors will be detected in the 2024 election but the possibility cannot be ruled out. If this option is selected and an error is detected in the audit sample, a second sample can be selected to reduce the confidence limit for the error rate.

Recommended: Sample of 1 in 90 scanned ballot papers (sample size of approximately 1000 papers)

This is the recommended option and will give Elections ACT a high degree of confidence that the audit sampling plan is highly defensible and withstands external scrutiny. The option can be implemented by selecting 1 in every 9 batches of ballot papers, and then selecting 1 in every 10 ballot papers from within the batch (i.e. 5 papers from each batch).

The total sample size under this option is 1022 if there are 92,000 paper ballots. The sample size will be over 1000 so long as at least 90,000 ballot papers are scanned in total. The audit sample size in the ACT for the 2022 senate election was 969, and so under this option a larger sample size will be used for the 2024 Legislative Assembly election so long as at least 87,210 paper ballots are submitted.

If this sampling option is used for the 2024 ACT Legislative assembly election, then if:

- *0 errors are detected in sample*, there is 95% confidence the error rate is lower than 0.29% and 99% confidence it is lower than 0.45%. In a population of 92,000 scanned voted this means 95% confidence there are not more than 267 erroneous ballot papers and 99% confidence there are not more than 409. As there are five electorates, this means the 95% limit is 53 erroneous papers and 99% limit of 82 erroneous papers in an electorate (for an electorate with a total of 18,400 scanned papers). The 99% limit of 82 errors is approximately 0.009 quotas.
- *1 error is detected in sample*, there is 95% confidence the error rate is lower than 0.46% and 99% confidence it is lower than 0.65%. In a population of 92,000 scanned voted this means 95% confidence there are not more than 423 erroneous ballot papers and 99% confidence there are not more than 591. The 99% confidence limit equates to 118 errors within an electorate where 18,400 ballot paper are scanned, or approximately 0.013 quotas.
- *2 errors are detected in sample*, there is 95% confidence the error rate is lower than 0.62% and 99% confidence it is lower than 0.82%. In a population of 92,000 scanned voted this means 95% confidence there are not more than 562 erroneous ballot papers and 99% confidence there are not more than 748. The 99% confidence limit equates to 150 errors within an electorate where 18,400 ballot paper are scanned, or approximately 0.017 quotas.

This option is recommended as it delivers an audit sample size that is similar to but slightly higher than the audit sample size used for ACT senate elections, and in the most likely outcome of no detected errors provides 99% confidence that the total number of errors is less than 0.01 the size of an average quota, as well as 99% confidence that the error rate is lower than the 0.45% error rate observed in the 2022 Australian Senate election[2] (for the whole of Australia). Even in the unlikely event of 2 errors being detected in the audit sample, there is still 99% confidence that the error rate sits below the error rate limits used for the AEC Senate audit design within the ACT.

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[2] In the 2022 Australian Senate election, the audit of scanned ballot papers estimated an error rate of 0.45% (Senate ballot paper sampling outcomes statement, Tom Rogers (Australian Electoral Commissioner), 7 July 2002)

It is also noted that the *Commonwealth Electoral Act 1918*[3] specifies that at least 1000 ballot papers be checked for an Australian Senate election within a senate electorate (i.e. a state or territory). While the act does not apply to ACT Legislative Assembly elections, the choice of an expected sample size of 1000 avoids potential criticism of auditing fewer ballots than is considered necessary for a Senate election.

Sample of 1 in 60 scanned ballot papers (sample size of approximately 1500 papers)

This option provides a higher level of confidence that election outcomes are not affected by scanning errors. The total sample size under this option of 1/60th the number of scanned ballots, or a sample size of 1533 if there are 92,000 paper ballots. The option can be implemented by selecting 1 in every 6 batches of ballot papers, and then selecting 1 in every 10 ballot papers from within the batch (i.e. 5 papers from each batch).

If this sampling option is used for the 2024 ACT Legislative assembly election, then if:

- *0 errors are detected in sample*, there is 95% confidence the error rate is lower than 0.20% and 99% confidence it is lower than 0.30%. In a population of 92,000 scanned voted this means 95% confidence there are not more than 177 erroneous ballot papers and 99% confidence there are not more than 271. As there are five electorates, this means the 95% limit is 35 erroneous papers and 99% limit of 54 erroneous papers in an electorate (for an electorate with a total of 18,400 scanned papers). The 99% limit of 54 errors is approximately 0.006 quotas.
- *1 error is detected in sample*, there is 95% confidence the error rate is lower than 0.31% and 99% confidence it is lower than 0.43%. In a population of 92,000 scanned voted this means 95% confidence there are not more than 281 erroneous ballot papers and 99% confidence there are not more than 393. The 99% confidence limit equates to 79 errors within an electorate where 18,400 ballot paper are scanned, or approximately 0.009 quotas.
- *2 errors are detected in sample*, there is 95% confidence the error rate is lower than 0.41% and 99% confidence it is lower than 0.55%. In a population of 92,000 scanned voted this means 95% confidence there are not more than 374 erroneous ballot papers and 99% confidence there are not more than 498. The 99% confidence limit equates to 100 errors within an electorate where 18,400 ballot paper are scanned, or approximately 0.011 quotas.

Under this option, even if 1 error is detected during the audit, there is still 99% confidence that the error rate is lower than the 0.45% error rate detected in the 2022 Australian senate election.

Calculation of a confidence interval for the error rate

A confidence interval can be calculated using the well-known Clopper-Pearson method[4], often referred to as the 'exact' method of calculating binomial confidence intervals. This method is discussed as an appropriate method to use in the paper by Blom et al. To calculate a one-sided 95% confidence interval for the error rate when x errors have been found in an audit sample of size n, determine the value of p for which $Pr(X \leq x) = 0.05$, where X is distributed as a Bin(n,p) distribution (a binomial distribution). The confidence interval is then (0,p); i.e. there is 95% confidence that the error rate is p or smaller.

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[3] Commonwealth Electoral Act 1918 Section 273AC (3) (b): Commonwealth Electoral Act 1918 (legislation.gov.au)

[4] Clopper, C.J. and Pearson, E.S. *The use of confidence or fiducial limits illustrated in the case of the binomial* Biometrika Vol 26 Issue 4 December 1934 pages 404-413: THE USE OF CONFIDENCE OR FIDUCIAL LIMITS ILLUSTRATED IN THE CASE OF THE BINOMIAL | Biometrika | Oxford Academic (oup.com)

This means that if the true error rate in the full population of scanned ballot papers is p, then there is only a 5% chance of observing as few as x errors in the audit sample. If the error rate is any value greater than p then the chance of observing as few as x errors in the audit sample is smaller than 5%.

The probability of observing x or fewer errors for a Bin(n,p) distribution can be calculated using the excel function BINOM.DIST.RANGE or by other statistical software or spreadsheet packages.

Appendix A: Performance of sampling schemes

The table below shows the performance of a 1 in 200, a 1 in 90 and a 1 in 60 sampling scheme for a population of 92,000 scanned ballots, in the case of 0, 1 or 2 detected errors within the audit sample. It gives both 95% and 99% confidence limits for the error rate and the total number of erroneous ballots in the population of 92,000.

For example, if a 1 in 90 sampling scheme is used and there are no errors detected in the audit sample, then there is 95% confidence that the error rate in the population does not exceed 0.29%, i.e. at most 0.29% of all scanned ballot papers have an error (267 erroneous ballots within the 92,000 population). If the actual error rate was greater than 0.29%, then the probability of selecting a sample of 1,022 and finding no errors in that sample is smaller than 5%.

Similarly, if a 1 in 90 sampling scheme is used and no errors are detected in the audit sample, there is 99% confidence that the error rate does not exceed 0.45%, i.e. there are at most 409 erroneous ballot papers in the population of 92,000.

Population size 92,000 scanned ballots			**Number of errors detected in sample**											
			0				**1**				**2**			
			95% upper bound for …		99% upper bound for …		95% upper bound for …		99% upper bound for …		95% upper bound for …		99% upper bound for …	
		Sample size	error rate	no. of errors	error rate	no. of errors	error rate	no. of errors	error rate	no. of errors	error rate	no. of errors	error rate	no. of errors
Sampling rate	**1 in 200**	460	0.65%	595	1.00%	913	1.03%	942	1.44%	1,315	1.36%	1,250	1.82%	1,663
	1 in 90	1,022	0.29%	267	0.45%	409	0.46%	423	0.65%	591	0.62%	562	0.82%	748
	1 in 60	1,533	0.20%	177	0.30%	271	0.31%	281	0.43%	393	0.41%	374	0.55%	498